Harry M. Markowitz is credited with introducing new concepts of risk measurement and their application to the selection of portfolios. He started with the idea of risk aversion of average investors and their desire to maximise the expected return with the least risk. Markowitz model is thus a theoretical framework for analysis of risk and return and their inter-relationships. He used the statistical analysis for measurement of risk and mathematical programming for selection of assets in a portfolio in an efficient manner. His framework led to the concept of efficient portfolios. An efficient portfolio is expected to yield the highest return for a given level of risk or lowest risk for a given level of return.

Markowitz generated a number of portfolios within a given amount of money or wealth and given preferences of investors for risk and return. Individuals vary widely in their risk tolerance and asset preferences. Their means, expenditures and investment requirements vary from individual to individual. Given the preferences, the portfolio selection is not a simple choice of anyone security or securities, but a right combination of securities. Markowitz emphasised that quality of a portfolio will be different from the quality of individual assets within it. Thus, the combined risk of two assets taken separately is not the same risk of two assets together. Thus, two securities of TIS CO do not have the same risk as one security of TIS CO and one of Reliance.

Risk and Reward are two aspects of investment considered by investors. The expected return may vary depending on the assumptions. Risk index is measured by the variance or the distribution around the mean, its range etc., which are in statistical terms called variance and covariance. The qualification of risk and the need for optimisation of return with lowest risk are the contributions of Markowitz. This led to what is called the Modern Portfolio Theory, which emphasises the trade off between risk and return. If the investor wants a higher return, he has to take higher risk. But he prefers a high return but a low risk and hence the need for a trade off.

A portfolio of assets involves the selection of securities. A combination of assets or securities is called a portfolio. Each individual investor puts his wealth in a combination of assets depending on his wealth, income and his preferences. The traditional theory of portfolio postulates that selection of assets should be based on lowest risk, as measured by its standard deviation from the mean of expected returns.

The greater the variability of returns, the greater is the risk. Thus, the investor chooses assets with the lowest variability of returns. Taking the return as the appreciation in the share price, (if TELCO shares) price varies from Rs. 338 to Rs. 580 (with variability of 72%) and Colgate from Rs. 218 to Rs. 315 (with a variability of 44%) during a time period, the investor chooses the Colgate as a less risky share.

As against this Traditional Theory that standard deviation measures the variability of return and risk is indicated by the variability, and that the choice depends on the securities with lower variability, the modem Portfolio Theory emphasises the need for maximisation of returns through a combination of securities, whose total variability is lower. The risk of each security is different from that of others and by a proper combination of securities, called diversification, one can arrive at a combination wherein the risk of one is offset partly or fully by that of the other.

In other words, the variability of each security and covariance for their returns reflected through their inter-relationships should be taken into account. Thus, as per the Modern Portfolio Theory, expected returns, the variance of these returns and covariance of the returns of the securities within the portfolio arc to be considered for the choice of a portfolio. A portfolio is said to be efficient, if it is expected to yield the highest return possible for the lowest risk or a given level of risk. A set of efficient portfolios can be generated by using the above process of combining various securities whose combined risk is lowest for a given level of return for the same amount of investment, the investor is capable of. The theory of Markowitz, as stated above is based on a number of assumptions.

**Assumptions of Markowitz Theory**

The Modern Portfolio Theory of Markowitz is based on the following assumptions:

1. Investors are rational and behave in a manner as to maximise their utility with a given level of income or money.

2. Investors have free access to fair and correct information on the returns and risk.

3. The markets are efficient and absorb the information quickly and perfectly.

4. Investors are risk averse and try to minimise the risk and maximise return.

5. Investors base decisions on expected returns and variance or standard deviation of these returns from the mean.

6. Investors prefer higher returns to lower returns for a given level of risk.

A portfolio of assets under the above assumptions for a given level of risk. Other assets or portfolio of assets offers a higher expected return with the same or lower risk or lower risk with the same or higher expected return. Diversification of securities is one method by which the above objectives can be secured. The unsystematic and company related risk can be secured. The unsystematic and company related risk can be reduced by diversification into various securities and assets whose variability is different and offsetting or put in different words which are negatively correlated or not correlated at all.

**Markowitz Diversification**

Markowitz postulated that diversification should not only aim at reducing the risk of a security by reducing its variability or
standard deviation, but by reducing the covariance or interactive risk of two or more securities in a portfolio. As by combination of different securities, it is theoretically possible to have a range of risk varying from zero to infinity.

Markowitz theory of portfolio diversification attaches importance to standard deviation, to reduce it to zero, if possible, covariance to have as much as possible negative interactive effect among the securities within the portfolio and coefficient of correlation to have -1 (negative) so that the overall risk of the portfolio as a whole is nil or negligible. Then the securities have to be combined in a manner that standard deviation is zero, as shown in the example below. Possible combinations of securities (1) and (2):

<table>
<thead>
<tr>
<th>Security (1)</th>
<th>Security (2)</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>20</td>
<td>0.8</td>
</tr>
<tr>
<td>70</td>
<td>30</td>
<td>0.4</td>
</tr>
<tr>
<td>66</td>
<td>34</td>
<td>0.0</td>
</tr>
<tr>
<td>20</td>
<td>80</td>
<td>0.8</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
<td>0.9</td>
</tr>
</tbody>
</table>

In the example, if 2/3rds arc invested in security (1) and 1/3rd in security

\[
\text{S.D.} = \frac{\text{S.D.}}{\text{Mean}} \quad \text{is the lowest.}
\]

The standard deviation of the portfolio determines the deviation of the returns and correlation coefficient of the proportion of securities in the portfolio, invested. The equation is

\[
\sigma_p = \text{portfolio variance} \\
\sigma_p = \text{Standard deviation of portfolio} \\
\sigma_i = \text{standard deviation of } i \\
\sigma_j = \text{standard deviation of } j \\
N = \text{number of securities.}
\]

**Problem**

Given the following example, find out the expected Risk of the portfolio

<table>
<thead>
<tr>
<th>Security</th>
<th>Expected Returns</th>
<th>Proportion % Invested (%)</th>
<th>SD (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.C.M.</td>
<td>10</td>
<td>20</td>
<td>0.2</td>
</tr>
<tr>
<td>Escorts</td>
<td>15</td>
<td>20</td>
<td>0.3</td>
</tr>
<tr>
<td>TSOCO</td>
<td>20</td>
<td>60</td>
<td>0.5</td>
</tr>
</tbody>
</table>

\[
\text{SD (Standard deviation) and coefficient of correlation is}
\]

\[
r_{ij} = 0.5 \quad (r_i \text{ with respect to } r_j)
\]

\[
r_{ij} = 0.1 \quad (r_i \text{ with respect to } r_j)
\]

\[
r_{ij} = -0.3 \quad (r_i \text{ with respect to } r_j)
\]

then

\[
\sigma_p^2 = \sigma_i^2 q_i^2 + \sigma_j^2 q_j^2 + 2 \rho_{ij} \sigma_i \sigma_j q_i q_j + 2 \sigma_i \sigma_j q_i q_j + 2 \sigma_i \sigma_j q_i q_j
\]

\[
= 0.016 + 0.036 + 0.09 + 0.0024 + 0.0108 + 0.0024
\]

\[
= 0.1 - 0.0108 = 0.0892
\]

\[
\sigma_p^2 = 0.0892
\]

\[
s_p = 0.0892 = 0.299 = +0.30 \text{ Portfolio Risk}
\]

**Parameters of Markowitz Diversification**

Based on his research, Markowitz has set out guidelines for diversification on the basis of the attitude of investors towards risk and return and on a proper quantification of risk. The investments have different types of risk characteristics, some caused systematic and market related risks and the other called unsystematic or company related risks. Markowitz diversification involves a proper number of securities, not too few or not too many which have no correlation or negative correlation. The proper choice of companies, securities, or assets whose return are not correlated and whose risks are mutually offsetting to reduce the overall risk.

For building up the efficient set of portfolio, as laid down by Markowitz, we need to look into these important parameters.

1. Expected return.
2. Variability of returns as measured by standard deviation from the mean.
3. Covariance or variance of one asset return to other asset returns.

In general the higher the expected return, the lower is the standard deviation or variance and lower is the correlation the better will be the security for investor choice. Whatever is the risk of the individual securities in isolation, the total risk of the portfolio of all securities may be lower, if the covariance of their returns is negative or negligible.

**Criteria of Dominance**

Dominance refers to the superiority of one portfolio over the other. A Set can dominate over the other, if with the same return, the risk is lower or with the same risk, the return is higher. Dominance principle involves the trade off between risk and return.

For two security portfolio, minimise the portfolio risk by the equation

\[
\text{Op} = \text{Wa} \sigma_a^2 + \text{Wb} \sigma_b^2 + 2 \rho_{ab} \sigma_a \sigma_b + 2 \text{Wa} \sigma_a \rho_{ab} \sigma_b + 2 \text{Wb} \sigma_b \rho_{ab} \sigma_a
\]

\[
\text{E (Rp)} = \text{WaE (Ra)} + \text{WbE (Rb)}
\]
R refers to returns and E (Rp) is the expected returns. \( \sigma \) is the standard deviation. \( W \) refers to the proportion invested in each security. \( \sigma_a \) and \( \sigma_b \) are the standard deviation of \( a \) and \( b \) securities and \( \sigma_{ab} \) is the covariance or interrelations of the security returns.

The above concepts are used in the calculation of expected returns, mean standard deviation as a measure of risk and covariance as a measure of inter-relations of one security return with another.

Markowitz Model Risk is discussed here in terms of a portfolio of assets.

As referred to earlier, any investment risk is the variability of return on a stock, assets or a portfolio. It is measured by standard deviation of the return over the Mean for a number of observations.

**Types of Risk**

**Summary**

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic</td>
<td>Unsystomatic</td>
</tr>
<tr>
<td>(Market)</td>
<td>(Company Risk)</td>
</tr>
<tr>
<td>Examples</td>
<td>Examples</td>
</tr>
<tr>
<td>Interest Rate Risk</td>
<td>Labour Troubles</td>
</tr>
<tr>
<td>Market Risk</td>
<td>Liquidity Problems</td>
</tr>
<tr>
<td>Inflation Risk</td>
<td>Raw Materials Risks</td>
</tr>
<tr>
<td>Demand and Government Policy</td>
<td>Financial Risks</td>
</tr>
<tr>
<td>International Factors</td>
<td>Management Problems</td>
</tr>
</tbody>
</table>

**Measurement of Risk (Example)**

Standard deviation to be calculated: Average in Mean

Observations: 10% - 5% 20% 35% - 10% = 10% will be their Mean.

<table>
<thead>
<tr>
<th>Deviation from the Mean</th>
<th>Square of Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-10 = 0</td>
<td>0</td>
</tr>
<tr>
<td>-5-15 = 15</td>
<td>225</td>
</tr>
<tr>
<td>+20-10 = 10</td>
<td>100</td>
</tr>
<tr>
<td>+35-10 =25</td>
<td>625</td>
</tr>
<tr>
<td>-10-10 =20</td>
<td>400</td>
</tr>
</tbody>
</table>

\[ \Sigma d^2 = 1350 \]

\[ \text{Standard Deviation} = \sqrt{\frac{\Sigma d^2}{6}} = \sqrt{\frac{1350}{5}} = 16.43 \]

<table>
<thead>
<tr>
<th>Stock x</th>
<th>Return</th>
<th>Expected Return</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>9</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>9</td>
<td>+4</td>
<td></td>
</tr>
</tbody>
</table>

Product \(-2 \times 4 = -8\)

<table>
<thead>
<tr>
<th>Stock y</th>
<th>Return</th>
<th>Expected Return</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>9</td>
<td>-4</td>
<td></td>
</tr>
</tbody>
</table>

Product = -8

Where, \( N = 2 \). The covariance equation is as follows:

\[ \text{Cov} = \frac{1}{2} \left[ (7-9)(13-9) + (11-9) (5-9) \right] \]

\[ = \frac{1}{2} \left[ (-2)(-4) + (-2)(-4) \right] = \frac{-16}{2} = -8 \]

The coefficient of correlation between two securities is \(-1.0\), it is perfect negative correlation. If its is \(+1.0\) it is perfect positive correlation. If the coefficient is ‘0’ then the returns are said to be independent. To sum up, correlation between two securities depend (a) on covariance between them, and (b) the standard deviation of each.

In Markowitz Model, we need to have inputs of expected return, risks measured by standard deviation of returns and the covariance between the returns on assets considered.
Portfolio Management (Summary)

Q. What is Portfolio Management.
A. Management of Large Investible Funds with a view to maximising return and minimizing risk.

Q. Investments are generally risky: the higher the risk, the higher the return. How to manage this trade-off between Risk and Return?
A. Efficient portfolios are those with minimum risk for a given expected return. For a planned return risk can be minimized by using the concept of Beta for systematic Risk and diversification for Unsystematic Risk.

Two Models Compared

<table>
<thead>
<tr>
<th>Markowitz Model</th>
<th>Utility of a portfolio is risk adjusted return. It is equal to portfolio return minus risk penalty. Where Risk Penalty = Risk Squared / Risk Tolerance.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sharpe Model</td>
<td>Optimal portfolio is set up by using the single index model of Sharpe. The desirability of any stock is directly related to its excess return to beta ratio, namely: Sharpe Index = (RJ - RF) / BJ where RJ is expected return, BJ is the Beta relating the stock to the market return. Then rank all the stock in their order of the index value.</td>
</tr>
</tbody>
</table>

In Sharpe model the return on any stock depends on some constant (alpha) called Alpha plus coefficient called (Beta) times the value of stock Index (I), plus a random component.

Share Model Equation was set as: 

\[ R_j = \alpha_j + \beta_j I + e_j \]

\[ R_j \] = Expected return on security J
\[ \alpha_j \] = Intercept of a straight line or alpha coefficient.
\[ \beta_j \] = Beta coefficient is the slope of straight line (Regression line)
\[ I \] = Expected return on Index of the market.
\[ e_j \] = Error term with a mean of zero and standard deviation which is constant.

Alpha (\( \alpha \)) is really the value of Y is the equation when the value of x is zero. The return on the stock in relation to the return on the market Index, namely, Beta is a measure of the systematic risk of the market. The error term in the above equation explains the unsystematic risk.

\[ Y = \alpha + \beta X + e \]

\[ Y = 0.91 + 0.93X, \text{ where } \beta = 0.93 \times \text{Market Return} \]

\[ Y = \text{Scrip Return} \]
\[ \alpha = \text{Constant} = 0.91 \]

Examples of Beta calculated:

- Reliance - 1.955
- ACC - 0.931
- Telco - 1.153
- Tisco - 1.342
- Colgate - 0.946
- Tata Tea - 0.951

Source: Some issues of Journal of chartered Financial Analyst (ICFAI)

On the basis of the above estimates of the stocks Alpha, Beta and expected return and Residual variance (data derived from the above formulas), one can construct a series of efficient portfolios by using a computer programmer. This will give out the series of corner portfolios and the line connecting them is the Efficient frontier line, in the shape Model.

The chart below represents the corner portfolio sets, indicating the efficient portfolio.
This is for calculating the expected return on portfolio where N = total number of stocks, X, is the proportion devoted to stock, i - R, is the Beta on stock i, I is market index return and is the same for all stocks, estimated.

For Portfolio Variance

\[ \sigma_p^2 = \text{Variation of portfolio return.} \]

\[ \sigma_{p,2}^2 = \text{expected variance of Index (Market)} \]

\[ \sigma_{c,2}^2 = \text{Variance in security's return not caused by its relationship to the index.} \]

**Arbitrage Pricing Theory**

**Introduction**

Like the capital Assets Pricing Model (CAPM), Arbitrage Pricing Theory (APT) is an equilibrium model of asset pricing but assumes that the returns are generated by factor Model. Its assumption vis-a-vis those CAPM are set out first:

<table>
<thead>
<tr>
<th>APT</th>
<th>CAPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investors do not look at expected and standard deviations. Based on the law of one price, if the price of an asset is different markets, arbitrage brings them to the same price. Investors prefer higher wealth/returns to lower wealth. APT is based on the return generated by factor models.</td>
<td>Investors look at the expected returns and accompanying risk measured by standard deviation. Investors are risk averse and risk-return analysis is necessary. Investors maximise wealth for a given level of risk.</td>
</tr>
</tbody>
</table>

**Asset selection in the Above Model**

Investment strategies of many types can also be selected under this model. If there are many securities to be selected, and a fixed amount to be invested, the investor can choose in a manner that he can aim at a zero non-factor risk (ei=0). This is possible by combining securities to hedge out the sensitivity of a portfolio to all but one factor.

An example will explain this. Let there are be three securities A, B and C with the following securities:

<table>
<thead>
<tr>
<th>Security</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-0.40</td>
<td>1.75</td>
</tr>
<tr>
<td>B</td>
<td>1.60</td>
<td>-0.75</td>
</tr>
<tr>
<td>C</td>
<td>0.67</td>
<td>-0.25</td>
</tr>
</tbody>
</table>

If he has Rs. 10000, he invests Rs. 300 in Security A, Rs. 700 in Security B and nil in security C, with proportions being 0.3 in A, 0.7 in B and 0 in C.

It will be seen from the equations below that the sensitivity to factors 1, 2 will be 1.0 and 0 respectively.

\[
\begin{align*}
\text{hp}_1 &= (-0.40 \times 0.3) + (1.60 \times 0.7) + (0.67 \times 0) \\
&= (-0.12) + (1.12) + (0) = +1.0 \\
\text{hp}_2 &= (1.75 \times 0.3) + (-0.75 \times 0.7) + (-0.25 \times 0) \\
&= (0.525) + (-0.525) = 0
\end{align*}
\]

In the above fashion it would be possible theoretically, although nor in practice to create “pure factor” portfolios that are sensitive to only one factor and have insignificant non-factor risk. But in practice only impair factor portfolios can be created.

**Components of Expected Returns**

It is convenient to break up the expected return into two parts: (i) risk free rate of return, and (ii) the rest in the following equation, \( r_f \) is the risk free return and \( y \) is the expected premium return per unit of sensitivity to the factor for Portfolio.

\[ r_p = r_f + y \]

Thus, the investor by splitting his funds among risk free portfolios and pure factor portfolios, it is possible for him to form a portfolio with almost any sensitivity to each factor. Although theory claims that the non-factor risk can be reduced to zero, it is not possible in real life. Therefore, in practical investment or in portfolio operations, it is better to combine the Capital Asset pricing theory and the APT Model. Most investors prefer, no doubt higher levels of expected return and dislike higher levels of risk. The fact is that there is a trade off between them, which is not considered by the APT Model.

Synthesis of CAPM and APT is therefore more realistic.

Beta coefficients can be used to reflect the risk factors and factor sensitivities can also be taken into account to arrive at the expected returns. Thus, if the returns are generated by two factor model, the Beta coefficient of a security will be related to its sensitiveness to the factors and factor Betas can be taken to reflect the different sensitivities of different factors.

It will be seen that \( \beta_1 \) and \( \beta_2 \) are constants as they do not vary from one security to another; the Beta conflict of a security is a function of its sensitiveness to the pervasive factors. Thus, by taking Betas of securities the question of sensitivity of security return to a factor is taken care of. \( j \) this, one can synthesize the PT Model and CAP Model in the empirical work.

**Empirical Testing of APT Model**

The CAPM as also the practical experience tells us that other things being equal, securities with large ex-ante betas will have relatively large expected returns. It does not mean that the actual ex-post returns will also be larger. But investment is made on expectation and hence the use of Betas, despite the fact that exact Betas may not really give an indication of actual returns in future.

Beta measurement is itself subject to limitations, as they change widely, with the number of years for which data are taken, the source of data, and the methods of compilation are subject to normal statistical limitations.
Using the actual data on Stock Price Index numbers and security prices of any Stock Exchange, one can compile Betas, the method of which has been explained earlier. Empirical studies done abroad on NYSE data (by Centre for Research in Security Prices (CRSP) at the University of Chicago) showed that the historical Beta values cannot be counted upon to predict the returns precisely. They are useful as some approximations and landmarks to go by.

In months, in which there were excess positive returns, the Beta factors were generally positive in the sense that stocks with historical Betas gave excess returns over the market. 111 periods when the excess returns were negative, d1C Beta factor was generally negative, in the sense that stocks with high historical betas, tended to under perform as compared to those with low-historical Betas.

In detailed tests of the original and Zero-beta CAPM, portfolio classes were used to examine the relationship between average returns and historical Betas. The graph below shows the actual relationship for the period 1938 to 1968. The vertical intercept, which corresponds to the zero beta return is 61% per month. While the average Treasury Bill rate, which corresponds to the risk free rate of interest, was only 0.13% per month. Assuming that Betas arc measured relative to a Stock index, they are surrogates for true Betas, and they support the thesis of CAPM to a substantial degree.

\[
x_1 = 1.8 \\
x_2 = 1.25 \\
x_3 = 0.50 \\
\]

\[
\beta_1 = 1.2 \\
\beta_2 = 1.5 \\
\beta_0 = -0.75 \\
\]

**Equation for APT Model**

\[
E(r_i) = \beta_{0} + \beta_{1} x_1 + \beta_{2} x_2 + \beta_{3} x_3 + \epsilon_i \\
\]

\[
E(r_i) = 7 + 1.8 (1.2) + 1.25 (1.5) + 0.50 (-0.75) \\
= 7 + 2.16 + 1.875 - 0.375 = 10.66 \\
\]

The expected rate of return of the stock is 10.66%.

**Question**

Calculate the equilibrium rate of return for the following three securities.

<table>
<thead>
<tr>
<th>Security</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>-0.5</td>
<td>0.75</td>
</tr>
<tr>
<td>C</td>
<td>0.75</td>
<td>1.30</td>
</tr>
</tbody>
</table>

**Answer**

Assume two factor Model as applicable which -

\[
E (r_i) = 4\% + 3\% b_{i1} + 5\% b_{i2} + 5\% b_{i3} \\
E (r_i) = 4\% + 3\% (-1.2) + 5\% (1.2) = 12.60 \\
E (r_i) = 4\% + 3\% (-0.5) + 5\% (0.75) = 6.25\% \\
E (r_i) = 4\% + 3\% (0.75) + 5\% (1.30) = 12.75\% \\
\]

In the Graph below, OM is the risk free return. Actual CAPM line is shown in the graph to vary from the Zero Beta line to a substantial extent.

**Sharpe Model**

Markotwiz Model had serious practical limitations due the rigours involved in compiling the expected returns, standard deviation, variance, covariance of each security to every other security in the portfolio. Sharpe model has simplified this process by relating the return in a security to a single Market index. Firstly, this will theoretically reflect all well traded securities in the market. Secondly, it will reduce and simplify the work involved in compiling elaborate matrices of variances as between individual securities.

If thus the market index is used as a surrogate for other individual securities in the portfolio, the relation of any individual security with the Market index can be represented in a Regression line or characteristic line. This is drawn below, with the excess return on the security on the y-axis and excess return on the Market Portfolio on the x-axis.

The equation of the characteristic line is

\[
R_i - R_f = \alpha + \beta (R_m - R_f) + \epsilon_i \\
\]

\( R_i \) is the holding period return on security i 
\( R_f \) is the risk less rate of interest
\( \alpha \) is the vertical intercept on y-axis representing the return on the security when only unsystematic risk is considered and systematic risk is measured by Beta. \( \epsilon_i \) is the residual component, not captured by the above variables.
Optimal Portfolio of Sharpe

This optimal portfolio of sharpe is called the single Index Model. The optimal portfolio is directly related to the Beta. If \( R_i \) is expected return on stock \( i \) and \( R_f \) is Risk free Rate, then the excess return = \( R_i - R_f \)

This has to be adjusted to \( \beta_i \), namely

\[
\frac{R_i - R_f}{\beta_i}
\]

which is the equation for ranking stocks in the order of their return adjusted for risk.

The method involves selecting a cut off rate for inclusion of securities in a portfolio. For this purpose, excess return to Beta ratio given above has to be calculated for each stock and rank them from highest to lowest.

\[
\frac{R_i - R_f}{\beta_i}
\]

Then only those securities which have greater than cut off point fixed in advance can be selected. The basis for finding the cut off Rate \( C \) is as follows :

**Basis for Cut off Rate**

For a portfolio of \( i \) stocks \( C_i \) is given by cut off rate.

**Example**

We have to see that for the optimum \( C_i \) that is \( C^* \) to be selected, the securities should have excess return to Betas above \( C_i \). Excess return to Beta ratio should be above \( C_i \) to be included in the portfolio, to be precise. This \( C \) is that point which shows the cut off point between those excess return to Beta ratios above. The calculation of \( C \) requires data, which are shown below :

\[
R_f = \text{Risk free Return} = 5\%
\]

<table>
<thead>
<tr>
<th>Security No.</th>
<th>Return Ri</th>
<th>Excess Return Ri</th>
<th>Beta Ri</th>
<th>Unsystematic Risk</th>
<th>Return to Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.0</td>
<td>10.0</td>
<td>1.0</td>
<td>50</td>
<td>10.0</td>
</tr>
<tr>
<td>2</td>
<td>37.0</td>
<td>12.0</td>
<td>1.5</td>
<td>46</td>
<td>8.0</td>
</tr>
<tr>
<td>3</td>
<td>12.0</td>
<td>7.0</td>
<td>1.0</td>
<td>20</td>
<td>7.0</td>
</tr>
<tr>
<td>4</td>
<td>17.0</td>
<td>12.0</td>
<td>2.0</td>
<td>10</td>
<td>6.0</td>
</tr>
<tr>
<td>5</td>
<td>11.0</td>
<td>6.0</td>
<td>1.0</td>
<td>46</td>
<td>6.0</td>
</tr>
</tbody>
</table>

All securities with excess return to Beta ratio above the cut off rate \( C^* \) say 3.0 in the above Table will be chosen in the Portfolio. The calculation of cut off point is also explained. In arriving at the optimal portfolio the emphasis of Sharpe Model is on Beta and on the Market Index. Sharpe's optimal portfolio would thus consist of those securities only which have excess return to Beta ratio above a cut off point.

By this method, selection of the portfolio has become easier due to the ranking of the securities in the order of their excess return and applying the yardstick of a required cut off point for selection of securities. That cut off point is related to the excess return to Beta rate on the one hand and variance of the market index and variance of the stock's movement which is related to the unsystematic risk, namely \( \sigma e_i^2 \).

**Question 1**

The return of Flex stock is related to factors 1 and 2 as given below

\[
(E) R_j = \lambda o + 0.6 \lambda_1 + 1.3 \lambda_2
\]

Where, 0.6 and 1.3 are sensitivity coefficients, \( \lambda \) risk premium is 6% and \( \lambda_2 \) is 3% and Risk free return \( (R_f) \) \( l \) = 7%

What is the stock's expected return ?

**Answer**

\[
(E) R_j = 0.07 + 0.6 \times 0.06 + 1.3 \times 0.03 = 0.145
\]

\[= 14.5\%
\]

**Question 2**

The reaction coefficient for two stocks and the market price \( l \) are given below

<table>
<thead>
<tr>
<th>Factor</th>
<th>( \lambda )</th>
<th>( b_A )</th>
<th>( b_B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.09</td>
<td>.5</td>
<td>.7</td>
</tr>
<tr>
<td>2</td>
<td>-0.03</td>
<td>.4</td>
<td>.8</td>
</tr>
<tr>
<td>3</td>
<td>.04</td>
<td>1.2</td>
<td>.2</td>
</tr>
</tbody>
</table>

Using APT model and risk free rate at 6% what is the expected return, if two stocks are equally weighted ?

**Answer**

\[
(E) R_A = 0.06 + .09 (.5) - .03 (.4) + .04 (1.2)
\]

\[= 0.06 + .045 - .012 + .048 = 0.141 = 14.1\%
\]

\[
(E) R_B = .06 + .09 (.7) - .03 (.8) + .04 (.2)
\]

\[= .06 + .063 - .024 + .008 = 0.107 = 10.7\%
\]

If they are equally weighted (0.5 and 0.5) Then

\[
(E) R_p = 0.5 (14.1) + 0.5 (10.7)
\]

\[= 7.05 + 5.35 = 12.4\%
\]

\[= 12.4\%
\]

There are two stock X and Y and three factors. Given the Risk free rate \( (R_f) \) at 6% \( \lambda \) corresponds to return on risk free asset or the \( \lambda \) in general reflects the market price of risk or expected excess return over the risk free return.

Given the earlier data on reaction coefficeints and if you are investing \( \frac{1}{3} \) in X and \( \frac{2}{3} \) in Y,

**Question 2**

What would be the expected return ?

**Answer**

\[
\Sigma R_j = \lambda + b_A F_1 + b_B F_2 + b_A F_3
\]

Using the above equations and ignoring the error them, we have

\[
R_X = 0.06 + 0.09 (.5) - .03 (.4) + .04 (1.2)
\]

\[= 0.06 + .045 - .012 = 0.141 = 14.1\%
\]

\[
R_Y = 0.06 + 0.09 (.7) - .03 (.8) + .04 (.2)
\]

\[= 0.06 + .063 - .024 + .008 = 0.107 = 10.7\%
\]

If we invest in the ratio of 1:2

then \( E R_p = 0.333 (14.1) + 0.666 (10.7) = \)
If we invest in the ratio 1:3 in the securities x and y
\[ ER_p = 0.25 (14.1) + 0.75 (10.7) = \]
\[ = 3.525 + 8.025 = 11.55\% \]

**Question 3**
Mr 'x' owns a portfolio with the following characteristics:

<table>
<thead>
<tr>
<th>Security</th>
<th>Factor Sensitivity</th>
<th>Factor Sensitivity</th>
<th>Proportion</th>
<th>Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.50</td>
<td>1.40</td>
<td>0.30</td>
<td>13%</td>
</tr>
<tr>
<td>B</td>
<td>1.60</td>
<td>0.30</td>
<td>0.20</td>
<td>18%</td>
</tr>
<tr>
<td>C</td>
<td>1.00</td>
<td>1.00</td>
<td>0.20</td>
<td>16%</td>
</tr>
<tr>
<td>D</td>
<td>1.00</td>
<td>1.30</td>
<td>0.20</td>
<td>12%</td>
</tr>
</tbody>
</table>

Assume that the returns are generated by a two factor model. Mr x decides to create an arbitrage Portfolio by increasing the holding of security 'B' by 0.05.

i. What must be the weights of the other three securities in his portfolio?

ii. What is the expected return on the arbitrage portfolio?

**Answer:**
1. \( Xa + Xb + Xc + Xd = 0 \)
2. \( Xa \times ba + Xb \times bb + Xc \times bc + Xd \times bd = 0 \)
3. \( 2.5xa + (1.6 \times 0.005) + 0.8 \times Xc + 2 \times d = 0 \)
   \( 1.4 xa + (0.9 \times 0.005) + 1 \times xc + 1.3 \times d = 0 \)

**Question 4**
Gopal holds portfolio of two companies A and B with the following details.

<table>
<thead>
<tr>
<th>Security</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Security Return</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Security Variance</td>
<td>0.0064</td>
<td>0.0016</td>
</tr>
<tr>
<td>Investment Proportion</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

Under the Markowitz Model what is the portfolio return and Portfolio Risk?

**Question 5**
Following data relates to two securities in the market i and j.

<table>
<thead>
<tr>
<th>Security</th>
<th>( E_r )</th>
<th>( \sigma )</th>
<th>( p_{ij} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>9</td>
<td>7.5%</td>
<td>-0.5</td>
</tr>
<tr>
<td>j</td>
<td>8</td>
<td>3.75%</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

Find out the minimum variance portfolio and compute its risk and return.

**Question**
What is optimum portfolio in choosing among the following securities assuming \( R_f = 5\% \) and \( \sigma_m = 10\% \) and \( R_m = 11\% \)